WHAT HAPPENS IN THE VICINITY OF THE SCHWARZSCHILD SPHERE WHEN NONZERO GRAVITON REST MASS IS PRESENT

A.A.Logunov, M.A.Mestvirishvili
Institute for High Energy Physics,
142284, Protvino, Moscow region, Russia

Abstract

In this paper a solution for a static spherically symmetric body is thoroughly considered in the framework of the Relativistic Theory of Gravitation. By the comparison of this solution with the Schwarzschild solution in General Relativity their substantial difference is established in the region close to the Schwarzschild sphere. Just this difference excludes the possibility of collapse to form "black holes".

The given problem was considered for the first time in the Relativistic Theory of Gravitation (RTG) in paper [1], where it was established that in vacuum the metric coefficient g_{00} of the effective Riemannian space was not equal to zero on the Schwarzschild sphere, whereas g_{11} had a pole. These changes which have arisen in the theory because of the graviton mass result in a "bounce" effect of the falling particles and light from a singularity on the Schwarzschild spere, and consequently, in the absence of "black holes".

Later in paper [2] an in-depth study of this problem in the RTG was conducted which updated a number of points, but at the same time showed, that the "bounce" took place close to the Schwarzschild sphere. In view of importance of this problem we again come back to its analysis with the purpose of showing in a simpler and clearer way that in that point in vacuum where the metric coefficient of effective Riemannian space g_{11} has a pole, another metric coefficient g_{00} will not vanish.

In RTG [3] the gravitational field is considered as a physical field in the Minkowski space. The source of this field is the universal conserved density of the energy-momentum tensor of the entire matter including the gravitational field. This circumstance results in the emerging of the effective Riemannian space because of the presence of the gravitational field. The motion of matter in the Minkowski space under the influence of the gravitational field proceeds in the same way as if it moved in the effective Riemannian space. The field approach to gravitation with necessity requires the introduction of the graviton rest mass.

In RTG, as opposed to the General Relativity Theory (GRT), the inertial reference frames are present and consequently the acceleration has an absolute meaning. The forces of inertia and gravity are separated, as they are of completely different nature. The Special Relativity Principle holds for all the physical fields, including the gravitational one. It follows from this theory that gravitational forces in the Newtonian approximation are the forces of attraction. Since a physical field can be described in one coordinate system, it means, that the effective Riemannian space has a simple topology and is set in one chart. In RTG the Mach Principle will be realised — an inertial reference frame is determined by the distribution of matter. In this theory the Correspondence Principle takes place: after switching off the gravitational field the curvature of space disappears, and we find ourselves in the Minkowski space in the coordinate system prescribed earlier.

The RTG equations look like

$$R^\mu_\nu - \frac{1}{2}\delta^\mu_\nu R + \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2 \left(\delta^\mu_\nu + g^{\mu\alpha}\gamma_{\alpha\nu} - \frac{1}{2}\delta^\mu_\nu g^{\alpha\beta}\gamma_{\alpha\beta}\right) = \kappa T^\mu_\nu, \qquad (1)$$

$$D_\mu \tilde{g}^{\mu\nu} = 0\ . \qquad (2)$$

Here $\tilde{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}$, $g = \det g_{\mu\nu}$, R^μ_ν is the Ricci tensor, $\kappa = \frac{8\pi G}{c^2}$, G is the gravitational constant, D_μ is the covariant derivative in the Minkowski space, $\gamma_{\mu\nu}(x)$ is the metric tensor of the Minkowski space in arbitrary curvilinear coordinates. Equations (1) and (2) are covariant under arbitrary coordinate transformations with a nonzero Jacobian. They are also Lorentz invariant under transformations from one inertial system in Galilean coordinates to another. Equations (2) eliminate representations corresponding to spins 1 and $0'$ for a tensor field, leaving only the representations with spins 2 and 0. The equations of motion of matter are the consequents of equations (1) and (2).

Let us determine now the gravitational field created by a spherically-symmetric static source. The general form of the interval of the effective Riemannian space for such source looks like

$$ds^2 = g_{00}dt^2 + 2g_{01}dtdr + g_{11}dr^2 + g_{22}d\Theta^2 + g_{33}d\Phi^2. \tag{3}$$

Let us introduce the notations

$$\begin{gathered}g_{00}(r) = U(r),\ g_{01}(r) = B(r),\ g_{11}(r) = -\left[V(r) - \frac{B^2(r)}{U(r)}\right],\\ g_{22}(r) = -W^2(r),\ g_{33}(r,\Theta) = -W^2(r)\sin^2\Theta.\end{gathered} \tag{4}$$

The components of the contravariant metric tensor are as follows:

$$\begin{gathered}g^{00}(r) = \frac{1}{U}\left(1 - \frac{B^2}{UV}\right),\ g^{01}(r) = -\frac{B}{UV},\ g^{11}(r) = -\frac{1}{V},\\ g^{22}(r) = -\frac{1}{W^2},\ g^{33}(r,\Theta) = -\frac{1}{W^2\sin^2\Theta}.\end{gathered} \tag{5}$$

The determinant of the metric tensor $g_{\mu\nu}$ is equal to

$$g = \det g_{\mu\nu} = -UVW^4\sin^2\Theta\ . \tag{6}$$

For the solution having a physical sense, the following condition should be satisfied:

$$g < 0\ . \tag{7}$$

For spherical coordinates g can be equal to zero only at a point $r = 0$. On the base of (5) and (6) we obtain the components of the metric tensor density

$$\tilde{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}. \tag{8}$$

They have the form

$$\tilde{g}^{00} = \frac{W^2}{\sqrt{UV}}\left(V - \frac{B^2}{U}\right)\sin\Theta,\ \tilde{g}^{01} = -\frac{BW^2}{\sqrt{UV}}\sin\Theta,\ \tilde{g}^{11} = -\sqrt{\frac{U}{V}}W^2\sin\Theta, \tag{9}$$

$$\tilde{g}^{22} = -\sqrt{UV}\sin\Theta,\ \tilde{g}^{33} = -\frac{\sqrt{UV}}{\sin\Theta}\ . \tag{9'}$$

All the consideration will be provided for an inertial system in spherical coordinates. The interval of the Minkowski space looks like

$$d\sigma^2 = dt^2 - dr^2 - r^2(d\Theta^2 + \sin^2\Theta d\Phi^2)\ . \tag{10}$$

Nonzero Christoffel symbols of the Minkowski space defined by the following formula

$$\gamma^{\lambda}_{\mu\nu} = \frac{1}{2}\gamma^{\lambda\sigma}(\partial_\mu\gamma_{\sigma\nu} + \partial_\nu\gamma_{\sigma\mu} - \partial_\sigma\gamma_{\mu\nu}) \tag{11}$$

are equal to

$$\gamma^1_{22} = -r,\ \gamma^1_{33} = -r\sin^2\Theta,\ \gamma^2_{12} = \gamma^3_{13} = \frac{1}{r},\ \gamma^2_{33} = -\sin\Theta\cos\Theta,\ \gamma^3_{23} = \cot\Theta\ . \tag{12}$$

Let us write equations (2) in the extended form

$$D_\mu\tilde{g}^{\mu\nu} = \partial_\mu\tilde{g}^{\mu\nu} + \gamma^{\nu}_{\lambda\sigma}\tilde{g}^{\lambda\sigma} = 0\ . \tag{13}$$

In Galilean coordinates of the Minkowski space they look like

$$\partial_\mu\tilde{g}^{\mu\nu} = 0\ . \tag{14}$$

In the case of a static gravitational field we have from (14)

$$\partial_i\tilde{g}^{i\nu} = 0,\ i = 1, 2, 3\ . \tag{15}$$

By using the tensor transformation law it is possible to express components $\tilde{g}^{i0}$ in Cartesian coordinates through components in spherical coordinates

$$\tilde{g}^{i0} = -\frac{BW^2}{\sqrt{UV}}\cdot\frac{x^i}{r^3}\ ,\ \sqrt{-g} = \frac{1}{r^2}\sqrt{UV}W^2. \tag{16}$$

Here x^i are spatial Cartesian coordinates. Supposing in (15) $\nu = 0$ and integrating over a spherical volume after applying the Gauss-Ostrogradskii theorem, we get the integral over a spherical surface

$$\oint\tilde{g}^{i0}ds_i = -\frac{BW^2}{r^3\sqrt{UV}}\oint(\vec{x}d\vec{s}) = 0\ . \tag{17}$$

Taking into consideration the equality

$$\oint(\vec{x}d\vec{s}) = 4\pi r^3, \tag{18}$$

we get

$$\frac{BW^2}{\sqrt{UV}} = 0 \; . \tag{19}$$

As equation (14) is fair both inside matter, and outside of it equation (19) should be true for any value of r. But as due to eqution (7) U, V and W cannot be equal to zero everywhere, it follows from (19) that

$$B = 0 \; . \tag{20}$$

Interval (3) of the effective Riemannian spaces becomes

$$ds^2 = Udt^2 - Vdr^2 - W^2(d\Theta^2 + \sin^2\Theta d\Phi^2) \; . \tag{21}$$

From equation (20) it follows, that there is no static solution for the Hilbert-Einstein equations in harmonic coordinates which would have in the interval expression the term like

$$B(r)dtdr \; . \tag{22}$$

The energy-momentum tensor of matter looks like

$$T^\mu_\nu = \left(\rho + \frac{p}{c^2}\right) v^\mu v_\nu - \delta^\mu_\nu \cdot \frac{p}{c^2} \; . \tag{23}$$

In expression (23) ρ is the mass density of matter, p is the isotropic pressure, and

$$v^\mu = \frac{dx^\mu}{ds} \tag{24}$$

is 4-velocity that meets the condition

$$g_{\mu\nu} v^\mu v^\nu = 1 \; . \tag{25}$$

From equations (1) and (2) it follows

$$\nabla_\mu T^\mu_\nu = 0 \; , \tag{26}$$

where ∇_μ is the covariant derivative in the effective Riemannian space with a metric tensor $g_{\mu\nu}$. In case of a static body

$$v^i = 0, \;\; i = 1,2,3; \;\; v^0 = \frac{1}{\sqrt{U}} \; , \tag{27}$$

and consequently

$$T_0^0=\rho(r),\ \ T_1^1=T_2^2=T_3^3=-\frac{p(r)}{c^2}\ ,\ \ T_\nu^\mu=0,\ \ \mu\neq\nu\ . \tag{28}$$

For interval (21) the nonzero Christoffel symbols are

$$\Gamma^0_{01}=\frac{1}{2U}\frac{dU}{dr}\ ,\ \Gamma^1_{00}=\frac{1}{2V}\frac{dU}{dr}\ ,\ \Gamma^1_{11}=\frac{1}{2V}\frac{dV}{dr}\ ,\ \Gamma^1_{22}=-\frac{W}{V}\frac{dW}{dr},$$
$$\Gamma^1_{33}=\sin^2\Theta\cdot\Gamma^1_{22}\ ,\ \Gamma^2_{12}=\Gamma^3_{13}=\frac{1}{W}\frac{dW}{dr}\ ,\ \ \Gamma^2_{33}=-\sin\Theta\cos\Theta\ ,\ \ \Gamma^3_{23}=\cot\Theta. \tag{29}$$

By using the following expression for the Ricci tensor

$$R_{\mu\nu}=\partial_\sigma\Gamma^\sigma_{\mu\nu}-\partial_\nu\Gamma^\sigma_{\mu\sigma}+\Gamma^\sigma_{\mu\nu}\Gamma^\lambda_{\sigma\lambda}-\Gamma^\sigma_{\mu\lambda}\Gamma^\lambda_{\sigma\nu}\ ,\ R^\mu_\nu=g^{\mu\lambda}R_{\lambda\nu} \tag{30}$$

and substituting into it the expressions for the Christoffel symbols from (29), it is possible to reduce equations (1) for functions U,V and W to the following form:

$$\frac{1}{W^2}-\frac{1}{VW^2}\left(\frac{dW}{dr}\right)^2-\frac{2}{VW}\frac{d^2W}{dr^2}-\frac{1}{W}\frac{dW}{dr}\frac{d}{dr}\left(\frac{1}{V}\right)+$$
$$+\frac{1}{2}\left(\frac{mc}{\hbar}\right)^2\left[1+\frac{1}{2}\left(\frac{1}{U}-\frac{1}{V}\right)-\frac{r^2}{W^2}\right]=\kappa\rho\ , \tag{31}$$

$$\frac{1}{W^2}-\frac{1}{VW^2}\left(\frac{dW}{dr}\right)^2-\frac{1}{UVW}\frac{dW}{dr}\cdot\frac{dU}{dr}+$$
$$+\frac{1}{2}\left(\frac{mc}{\hbar}\right)^2\left[1-\frac{1}{2}\left(\frac{1}{U}-\frac{1}{V}\right)-\frac{r^2}{W^2}\right]=-\kappa\frac{p}{c^2}\ , \tag{32}$$

$$-\frac{1}{VW}W''-\frac{1}{2UV}U''+\frac{1}{2WV^2}W'V'+\frac{1}{4VU^2}(U')^2+$$
$$+\frac{1}{4UV^2}U'V'-\frac{1}{2UVW}W'U'+\frac{1}{2}\left(\frac{mc}{\hbar}\right)^2\left[1-\frac{1}{2}\left(\frac{1}{U}+\frac{1}{V}\right)\right]=-\kappa\frac{p}{c^2}\ . \tag{33}$$

Equation (13) after taking into account (12), (9) and (20) is as follows:

$$\frac{d}{dr}\left(\sqrt{\frac{U}{V}}W^2\right)=2r\sqrt{UV}\ . \tag{34}$$

Let us remark that by virtue of the Bianchi identity and equation (2) one of equations (31-33) is a consequent of the others. Further we shall take equations (31), (32) and (34) as independent.

We shall write equation (26) in the extended form as

$$\nabla_\mu T^\mu_\nu \equiv \partial_\mu T^\mu_\nu + \Gamma^\mu_{\alpha\mu} T^\alpha_\nu - \Gamma^\alpha_{\mu\nu} T^\mu_\alpha = 0\,. \tag{35}$$

By using expressions (28) and (29) we obtain

$$\frac{1}{c^2}\cdot\frac{dp}{dr} = -\frac{\rho+\frac{p}{c^2}}{2U}\cdot\frac{dU}{dr}\,. \tag{36}$$

Taking into consideration identity

$$\frac{1}{W^2\left(\frac{dW}{dr}\right)}\cdot\frac{d}{dr}\cdot\left[\frac{W}{V}\left(\frac{dW}{dr}\right)^2\right] = \frac{1}{VW^2}\left(\frac{dW}{dr}\right)^2 + \frac{2}{VW}\cdot\frac{d^2W}{dr^2} + \frac{1}{W}\frac{dW}{dr}\frac{d}{dr}\left(\frac{1}{V}\right)\,, \tag{37}$$

equation (31) can be written in the following form

$$1 - \frac{d}{dW}\left[\frac{W}{V\left(\frac{dr}{dW}\right)^2}\right] + \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2\left[W^2 - r^2 + \frac{W^2}{2}\left(\frac{1}{U} - \frac{1}{V}\right)\right] = \kappa W^2 \rho\,. \tag{38}$$

Similarly we transform equation (32):

$$1 - \frac{W}{V\left(\frac{dr}{dW}\right)^2}\cdot\frac{d}{dW}\ln(UW) + \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2\left[W^2 - r^2 - \frac{1}{2}\left(\frac{1}{U} - \frac{1}{V}\right)\right] = -\kappa\frac{W^2 p}{c^2}\,. \tag{39}$$

We shall write Eqs. (34) and (36) as follows:

$$\frac{d}{dW}\left(W^2\sqrt{\frac{U}{V}}\right) = 2r\sqrt{UV}\frac{dr}{dW}\,. \tag{40}$$

$$\frac{1}{c^2}\cdot\frac{dp}{dW} = -\left(\rho + \frac{p}{c^2}\right)\frac{1}{2U}\cdot\frac{dU}{dW}\,. \tag{41}$$

Let us proceed to dimensionless variables in equations (38) – (41). Let l be the Schwarzshild radius of the source which has mass M

$$l = \frac{2GM}{c^2}\,. \tag{42}$$

Let us introduce new variables x and z which are equal to

$$W = lx, \quad r = lz. \tag{43}$$

Equations (38-41) become

$$1 - \frac{d}{dx}\left(\frac{x}{V\left(\frac{dz}{dx}\right)^2}\right) + \epsilon\left[x^2 - z^2 + \frac{1}{2}x^2\left(\frac{1}{U} - \frac{1}{V}\right)\right] = \tilde{\kappa}x^2\rho(x), \tag{38$'$}$$

$$1 - \frac{x}{V\left(\frac{dz}{dx}\right)^2}\frac{d}{dx}\ln(xU) + \epsilon\left[x^2 - z^2 - \frac{x^2}{2}\left(\frac{1}{U} - \frac{1}{V}\right)\right] = -\tilde{\kappa}\frac{x^2 p(x)}{c^2}\ , \tag{39$'$}$$

$$\frac{d}{dx}\left(x^2\sqrt{\frac{U}{V}}\right) = 2z\frac{dz}{dx}\sqrt{UV}\ , \tag{40$'$}$$

$$\frac{1}{c^2}\frac{dp}{dx} = \left(\rho + \frac{p}{c^2}\right)\frac{1}{2U}\frac{dU}{dx}\ . \tag{41$'$}$$

Here ϵ is a numerical constant which is equal to

$$\epsilon = \frac{1}{2}\left(\frac{2GMm}{\hbar c}\right)^2, \quad \tilde{\kappa} = \kappa l^2. \tag{44}$$

The sum and difference of equations (38$'$) and (39$'$) are

$$2 - \frac{d}{dx}\left[\frac{x}{V\left(\frac{dz}{dx}\right)^2}\right] - \frac{x}{V\left(\frac{dz}{dx}\right)^2}\frac{d}{dx}\ln(xU) + 2\epsilon(x^2 - z^2) = \tilde{\kappa}x^2\left(\rho - \frac{p}{c^2}\right)\ , \tag{45}$$

$$\frac{d}{dx}\left[\frac{x}{V\left(\frac{dz}{dx}\right)^2}\right] - \frac{x}{V\left(\frac{dz}{dx}\right)^2}\frac{d}{dx}\ln(xU) - \epsilon x^2\left(\frac{1}{U} - \frac{1}{V}\right) = -\tilde{\kappa}x^2\left(\rho + \frac{p}{c^2}\right)\ . \tag{46}$$

Let us introduce new functions A and η:

$$U = \frac{1}{x\eta A}, \quad V = \frac{x}{A\left(\frac{dz}{dx}\right)^2}\ . \tag{47}$$

In these new variables equation (45) becomes

$$A\frac{d\ln\eta}{dx} + 2 + 2\epsilon(x^2 - z^2) = \tilde{\kappa}x^2\left(\rho - \frac{p}{c^2}\right)\ . \tag{48}$$

Equation ($38'$) can be written in the following form:

$$\frac{dA}{dx} = 1 + \epsilon(x^2 - z^2) + \epsilon\frac{x^2}{2}\left(\frac{1}{U} - \frac{1}{V}\right) - \tilde{\kappa}\cdot x^2\rho(x) \ . \tag{49}$$

According to the causality condition (see Appendix L)

$$\gamma_{\mu\nu}U^{\mu}U^{\nu} = 0 \ , \tag{50}$$

$$g_{\mu\nu}U^{\mu}U^{\nu} \leq 0 \ , \tag{50$'$}$$

it is easy to establish the following inequality:

$$U \leq V \ . \tag{51}$$

In our problem it is possible to limit ourselves by values x and z from the following interval only:

$$0 \leq x \ll \frac{1}{\sqrt{2\epsilon}}, \ \ 0 \leq z \ll \frac{1}{\sqrt{2\epsilon}}. \tag{52}$$

These inequalities limit r, W from above by the value

$$r, W \ll \frac{\hbar}{mc} \ . \tag{53}$$

Under such a limitation equation (49) becomes

$$\frac{dA}{dx} = 1 + \epsilon\frac{x^2}{2}\left(\frac{1}{U} - \frac{1}{V}\right) - \tilde{\kappa}x^2\rho(x) \ . \tag{54}$$

Outside of the matter we have

$$\frac{dA}{dx} = 1 + \epsilon\frac{x^2}{2}\left(\frac{1}{U} - \frac{1}{V}\right) \ . \tag{55}$$

By virtue of the causality condition (51) the following inequality takes place outside of matter

$$\frac{dA}{dx} \geq 1. \tag{56}$$

Integrating (54) in an interval $(0, x)$ we get

$$A(x) = x + \frac{\epsilon}{2}\int\limits_0^x x'^2\left(\frac{1}{U} - \frac{1}{V}\right)dx' - \tilde{\kappa}\int\limits_0^x x'^2\rho(x')dx'. \tag{57}$$

In (57) $A(0)$ is trusted to be equal to zero, as if it was distinct from zero, the function $V(x)$ would become zero when x tends to zero, that is physically forbidden. On the base of relation (56) function $A(x)$ monotonically increases with x outside of matter, and therefore it can have only one root

$$A(x_1) = 0, \quad x_1 > x_0. \tag{58}$$

On the base of relation (57) we have

$$x_1 = 1 - \frac{\epsilon}{2}\int\limits_0^{x_1} x'^2 \left(\frac{1}{U} - \frac{1}{V}\right) dx'. \tag{59}$$

Here we take into account that by selecting l equal to (42)

$$\tilde{\kappa}\int\limits_0^{x_0} x'^2 \rho(x') dx' = 1.$$

The matter is concentrated in the sphere $0 \le x \le x_0$.

Because of a graviton mass, zero point of function A is shifted inside the Schwarzshild sphere. As at x tending to x_1, $V(x)$ is tending to infinity, since $A(x)$ is going to zero, there will be such neighborhood of a point x_1

$$x_1(1-\lambda_1) \le x \le x_1(1+\lambda_2), \quad \lambda_1 > 0, \lambda_2 > 0\ , \tag{60}$$

(λ_1 and λ_2 receive small fixed values), in which the following inequality will take place

$$\frac{1}{U} \gg \frac{1}{V}\ . \tag{61}$$

In this approximation we obtain

$$A(x) = x - x_1 + \frac{\epsilon}{2}\int\limits_{x_1}^{x} dx' x'^2 \frac{1}{U}\ . \tag{62}$$

Substituting U in the form given by relation (47) into this expression, we shall discover

$$A(x) = x - x_1 + \frac{\epsilon}{2}\int\limits_{x_1}^{x} dx' x'^3 \eta(x') A(x'). \tag{63}$$

If a range of x is in interval (60), then in the integrand it is possible to change x^3 for x_1^3:

$$A(x) = x - x_1 + \frac{\epsilon}{2} x_1^3 \int\limits_{x_1}^{x} \eta(x') A(x') dx'. \tag{64}$$

From here we get

$$\frac{dA}{dx} = 1 + \frac{\epsilon}{2} x_1^3 \eta(x) A(x). \tag{65}$$

In the approximation considered (52) equation (48) becomes

$$A \frac{d \ln \eta}{dx} + 2 = 0 \ . \tag{66}$$

Let us introduce a new function

$$f(x) = \frac{x_1^3}{2} \eta(x) A(x) \ . \tag{67}$$

Equation (65) becomes

$$\frac{dA}{dx} = 1 + \epsilon f(x) \ , \tag{68}$$

and equation (66) takes the following form

$$\frac{A}{f} \cdot \frac{df}{dx} - \frac{dA}{dx} = -2 \ . \tag{69}$$

From equations (68) and (69) we find

$$A(x) = -\frac{(1 - \epsilon f) f}{\left(\frac{df}{dx}\right)} \ . \tag{70}$$

From expression (67) we get

$$\eta(x) = -\frac{2 \frac{df}{dx}}{x_1^3 (1 - \epsilon f)} \ . \tag{71}$$

Substituting (70) and (71) into (47) we discover

$$U = \frac{x_1^3}{2xf}, \quad V = -\frac{x \frac{df}{dx}}{f(1 - \epsilon f) \left(\frac{dz}{dx}\right)^2} \ . \tag{72}$$

By using these expressions the determinant g can be written in the following form

$$g = \frac{x_1^3 \frac{df}{dx} x^4}{2f^2 \left(\frac{dz}{dx}\right)^2 (1-\epsilon f)} \sin^2\Theta < 0 \ . \tag{73}$$

For fulfilment of condition (7) it is necessary that expressions $\frac{df}{dx}$ and $(1-\epsilon f)$ have opposite signs. Substituting (70) into (68) we get

$$\frac{d}{dx}\ln\left|\frac{df}{dx}\right| - \frac{d}{dx}\ln|f(1-\epsilon f)| = \frac{1+\epsilon f}{f(1-\epsilon f)} \cdot \frac{df}{dx} \ . \tag{74}$$

From here we find

$$\frac{d}{dx}\ln\left|\frac{(1-\epsilon f)\frac{df}{dx}}{f^2}\right| = 0 \ . \tag{75}$$

Thus

$$\left|\frac{(1-\epsilon f)\frac{df}{dx}}{f^2}\right| = C_0 > 0 \ . \tag{76}$$

Taking into account that the values $(1-\epsilon f)$ and $\frac{df}{dx}$ should have opposite signs, we obtain

$$\frac{df}{dx} = -\frac{C_0 f^2}{(1-\epsilon f)} \ . \tag{77}$$

Substituting this expression in (70) we find

$$A(x) = \frac{(1-\epsilon f)^2}{C_0 f}, \ \ A(x_1) = 0 \quad \text{under} \quad f = \frac{1}{\epsilon} \ . \tag{78}$$

By taking into account (78) expression (47) for function V becomes

$$V = \frac{C_0 x f}{(1-\epsilon f)^2 \left(\frac{dz}{dx}\right)^2} \ . \tag{79}$$

Integrating (77) and allowing for (78) we get

$$C_0 \cdot (x - x_1) = \frac{1}{f} + \epsilon \ln \epsilon |f| - \epsilon \ . \tag{80}$$

Relation (80) is obtained in a range of values x determined by inequalities (60), however, it is also correct in the area where the influence of a graviton mass can be neglected.

According to (60) the range of $C_0(x-x_1)$ is confined to limits

$$-C_0x_1\lambda_1 \le C_0(x-x_1) \le C_0x_1\lambda_2, \tag{81}$$

if f is positive, it satisfies inequalities

$$\tilde{C} \le f \le \frac{1}{\epsilon}\,. \tag{82}$$

By using (80) and according to (81) we have

$$\frac{1}{f} + \epsilon \ln \epsilon f - \epsilon \le C_0x_1\lambda_2.$$

From here it is possible to find $\tilde{C}$:

$$\frac{1}{\tilde{C}} + \epsilon \ln \epsilon \tilde{C} - \epsilon = C_0x_1\lambda_2. \tag{83}$$

From expression (83) we can find an approximate value for $\tilde{C}$:

$$\tilde{C} = \frac{1}{C_0x_1\lambda_2}\,. \tag{84}$$

For negative values f to a point $x = x_1$ corresponds the value $|f|$, determined from the following equation

$$-\frac{1}{|f|} + \epsilon \ln \epsilon |f| - \epsilon = 0\,. \tag{85}$$

From here we get

$$|f| = \frac{a}{\epsilon}, \;\; \ln a = \frac{1+a}{a}\,. \tag{86}$$

According to (81) the following inequality should to be fulfilled

$$-C_0x_1\lambda_1 \le -\frac{1}{|f|} + \epsilon \ln \epsilon |f| - \epsilon\,. \tag{87}$$

From here it is possible to find the lower limit for $|f| = D$

$$-C_0x_1\lambda_1 = -\frac{1}{D} + \epsilon \ln \epsilon D - \epsilon\,. \tag{88}$$

From expression (88) we discover an approximate value for D

$$D = \frac{1}{C_0 x_1 \lambda_1} \ . \tag{89}$$

It means, that the value of $|f|$ fulfils the following inequality:

$$|f| \geq D = \frac{1}{C_0 x_1 \lambda_1} \ . \tag{89$'$}$$

Let us establish now the form of dependence of variable z of x. Substituting (47) in (40$'$) and allowing for (48), we get

$$A\frac{d}{dx}\left(x\frac{dz}{dx}\right) = 2z - x\frac{dz}{dx}\left[1 + \epsilon(x^2 - z^2) - \frac{1}{2}\tilde{\kappa}x^2\left(\rho - \frac{p}{c^2}\right)\right] \ . \tag{90}$$

In approximation (52) outside of matter equation (90) becomes

$$A\frac{d}{dx}\left(x\frac{dz}{dx}\right) + x\frac{dz}{dx} - 2z = 0 \ . \tag{91}$$

It is necessary for us to find the regular solution $z(x)$ of equation (91). In equation (91) we shall proceed from variable x to f. By using relation (80)

$$x = \frac{1}{C_0 f}[C_0 x_1 f + 1 - \epsilon f + \epsilon f \ln \epsilon |f|] \ , \tag{92}$$

and allowing for (65), (66) and (83), equation (91) can be presented in the following form

$$\frac{d^2 z}{df^2} + \frac{C_0 x f + \epsilon f - 1}{C_0 f^2 x} \cdot \frac{dz}{df} - \frac{2z}{C_0 f^3 x} = 0 \ . \tag{93}$$

By a straightforward substitution we can establish that the expression

$$z = \frac{x_1}{2} + \frac{1}{C_0 f}[1 - \epsilon f + \epsilon f \ln \epsilon |f|] \tag{94}$$

satisfies equation (93) up to the value

$$\epsilon\frac{(1 - \epsilon f + \ln \epsilon |f|)}{C_0^2 x f^3} \ , \tag{95}$$

which is extremely small in the neighborhood of the point x_1. From expressions (92) and (94) we find

$$z = x - \frac{x_1}{2} \; . \tag{96}$$

Allowing for this relation and also (79) and (72), we get

$$U = \frac{x_1^3}{2xf}, \;\; V = \frac{C_0 x f}{(1-\epsilon f)^2}. \tag{97}$$

For negative values f the causality condition (51) becomes

$$(2x^2 C_0 - \epsilon^2 x_1^3) - 2\epsilon x_1^3 |f| - x_1^3 \leq 0. \tag{98}$$

Inequality (98) is not valid, as it does not fulfil inequality (89′). Thus, the Principle of Causality is violated in the region of negative values of f. It means that in the area $x_1(1-\lambda_1) \leq x < x_1$ the solution has no physical sense. At $x_0 < x_1(1-\lambda_1)$ the situation arises, when the physical solution inside a body $0 \leq x \leq x_0$ cannot be sewed to the physical solution in the region $x > x_1$, as there is an intermediate region $x_1(1-\lambda_1) \leq x < x_1$, in which the solution does not satisfy the Causality Principle. It follows from here with necessity that $x_0 \geq x_1$. From the physical point of view it is necessary to eliminate also the equality $x_0 = x_1$, as the solution inside a body should continuously pass into the external solution. Therefore, the variable f takes only positive values, and x_0 cannot be less than x_1. For the values from the region $x \geq x_1(1+\lambda_2)$ it is possible to omit the terms with a small parameter ϵ in equations (38′) and (39′). Thus, we shall come to the external Schwarzshild solution

$$z_s = (x - \omega)\left[1 + \frac{b}{2\omega}\ln\frac{x-2\omega}{x}\right] \; , \tag{99}$$

$$V_s = \frac{x}{\left(\frac{dz}{dx}\right)^2 (x - 2\omega)} \; , U_s = \frac{x-2\omega}{x} \; . \tag{100}$$

Here " ω " and " b " are some constants, which are determined from the condition of sewing solutions (96), (97) with the solution (99), (100). The function z from (96) is equal to

$$z = x_1\left(\frac{1}{2} + \lambda_2\right) \; , \tag{101}$$

at the point $x = x_1(1+\lambda_2)$. At the same point z_s is equal to

$$z_s = [x_1(1+\lambda_2) - \omega]\left[1 + \frac{b}{2\omega}\ln\frac{x_1(1+\lambda_2) - 2\omega}{x_1(1+\lambda_2)}\right]. \tag{102}$$

From a sewing condition of (101) and (102) we find

$$\omega = \frac{x_1}{2}, \quad b = 0 \,. \tag{103}$$

The function U from (97) is equal to

$$U = \frac{x_1^3}{2x_1(1+\lambda_2)\tilde{C}} \,, \tag{104}$$

at the point $x = x_1(1+\lambda_2)$, as $\tilde{C}$, according to (84), is equal to

$$\tilde{C} = \frac{1}{C_0 x_1 \lambda_2} \,. \tag{105}$$

By substituting (105) into (104) we get

$$U = \frac{C_0 x_1^3 \lambda_2}{2(1+\lambda_2)} \,, \tag{106}$$

at the same point, with account for (103), U_s is equal to

$$U_s = \frac{\lambda_2}{1+\lambda_2} \,. \tag{107}$$

From a sewing condition of (106) and (107) we get

$$C_0 = \frac{2}{x_1^3} \,. \tag{108}$$

At the point $x = x_1(1+\lambda_2)$ the function V from (97) is equal to

$$V = C_0 x_1 (1+\lambda_1)\tilde{C} \,. \tag{109}$$

By substituting the value $\tilde{C}$ from (105) into (109) we obtain

$$V = \frac{1+\lambda_2}{\lambda_2} \,, \tag{110}$$

at the same point V_s, with account for (99) and (103), is equal to

$$V_s = \frac{1+\lambda_2}{\lambda_2} \ , \tag{111}$$

i.e. the solution for V is sewed to the solution for V_s.

Let us consider (92) for values ϵf, close to unity

$$f = \frac{1}{\epsilon\left(1+\frac{y}{\epsilon}\right)}, \quad \frac{y}{\epsilon} \ll 1. \tag{112}$$

By substituting this expression into (92) and expanding it over $\frac{y}{\epsilon}$, we obtain

$$y^2 = 2\epsilon C_0(x-x_1). \tag{113}$$

Inequality (112) tells us that the value $(x-x_1)=\delta \ll \epsilon$, i.e.

$$\frac{y}{\epsilon} = \sqrt{2C_0}\cdot\sqrt{\frac{x-x_1}{\epsilon}} \ll 1 \ . \tag{114}$$

By substituting (113) into (112), and then f into (97), we get the following expressions for U and V :

$$U = \frac{x_1^3[\epsilon+\sqrt{2\epsilon C_0(x-x_1)}]}{2x} \ , \quad V = \frac{x[\epsilon+\sqrt{2\epsilon C_0(x-x_1)}]}{2\epsilon(x-x_1)} \ . \tag{115}$$

From here we have in the region of variable x, satisfying inequality (114),

$$U = \frac{\epsilon x_1^3}{2x}, \quad V = \frac{x}{2(x-x_1)} \ . \tag{116}$$

We see, that the presence of a graviton mass essentially changes the nature of solution in the region close to the gravitational radius. In that point, where the function V, according to (116), has a pole, the function U is different from zero, whereas in the General Relativity Theory (GRT) it is equal to zero. Just by virtue of this circumstance the inevitable gravitational collapse arises, during which “black holes” appear in GRT. In RTG “black holes” are impossible.

If we take into account (42), (43), (96) and neglect the second term in (59), expressions (116) for U and V become

$$U = \left(\frac{GMm}{\hbar c}\right)^2, \quad V = \frac{1}{2}\cdot\frac{r+\frac{GM}{c^2}}{r-\frac{GM}{c^2}} \ , \tag{117}$$

which coincides with formulas (18) from paper [1]. Note, that the residue in the pole of the function V at $\epsilon \neq 0$ is equal to $\frac{GM}{c^2}$, whereas at $\epsilon = 0$ it is equal to $\frac{2GM}{c^2}$. This is due to the fact that in case $\epsilon = 0$ the pole of function V at the point $x = x_1$ arises because of the function f, which has a pole at this point , whereas at $\epsilon \neq 0$ it occurs because of the function $(1-\epsilon f)$, which one, according to (92), at the point $x = x_1$ comes into zero.

Let us compare now the nature of motion of test bodies in the effective Riemannian space with metric (117) and with the Schwarzshild metric. We write the interval (21) of the Riemannian space as follows:

$$ds^2 = Udt^2 - \tilde{V}dW^2 - W^2(d\Theta^2 + \sin^2\Theta d\Phi^2) \ . \tag{118}$$

Here $\tilde{V}$ is equal to

$$\tilde{V}(W) = V\left(\frac{dr}{dW}\right)^2 . \tag{119}$$

The motion of a test body takes place along a geodesic line of the Riemannian space

$$\frac{dv^\mu}{ds} + \Gamma^\mu_{\alpha\beta}v^\alpha v^\beta = 0 \ , \tag{120}$$

where

$$v^\mu = \frac{dx^\mu}{ds} \ , \tag{121}$$

the four-vector of velocity v^μ meets the following condition:

$$g_{\mu\nu}v^\mu v^\nu = 1 \ . \tag{122}$$

Let us consider a radial motion, when

$$v^\Theta = v^\Phi = 0 \ . \tag{123}$$

By taking into account (29), from equation (120) we find

$$\frac{dv^0}{ds} + \frac{1}{U}\cdot\frac{dU}{dW}v^0v^1 = 0 \ , \tag{124}$$

where

$$v^1 = \frac{dW}{ds} \ . \tag{125}$$

From equation (124) we get

$$\frac{d}{dW}\ln(v^0U) = 0 \ . \tag{126}$$

From here we have

$$v^0 = \frac{dx^0}{ds} = \frac{U_0}{U} \ , \tag{127}$$

where U_0 is a constant of integrating.

Taking into account (127) we see that condition (122) for radial motion becomes

$$\frac{U_0^2}{U} - 1 = \tilde{V} \cdot \left(\frac{dW}{ds}\right)^2 . \tag{128}$$

If we accept, the speed of a falling test body at infinity being equal to zero, we shall get $U_0 = 1$. From (128) is follows

$$\frac{dW}{ds} = -\sqrt{\frac{1-U}{U\tilde{V}}} \ . \tag{129}$$

Taking into consideration (79), (96), (97) and (108), we have

$$U = \frac{x_1^3}{2xf}, \ \ \tilde{V} = \frac{2xf}{x_1^3(1-\epsilon f)^2} \ .$$

Substituting these expressions in (129) we get

$$\frac{dW}{ds} = -\sqrt{1-U}(1-\epsilon f) \ . \tag{130}$$

By using (108), (112) and (113) in the neighborhood of the point x_1 we have

$$\frac{dW}{ds} = -\frac{2}{x_1}\sqrt{\frac{x-x_1}{\epsilon x_1}} \tag{131}$$

Passing from a variable x to W, according to (43) and taking into account (44), we obtain

$$\frac{dW}{ds} = -\frac{\hbar c^2}{mGM}\sqrt{\frac{W}{GM}\left(1-\frac{2GM}{c^2 W}\right)} \ . \tag{132}$$

It is apparent from here that there is a turning point. By differentiating (132) on s we get

$$\frac{d^2W}{ds^2} = \frac{1}{2GM}\left(\frac{\hbar c^2}{mGM}\right)^2 . \tag{133}$$

In the turning point the acceleration (133) is rather great, and it is positive, i.e. repulsing takes place. By integrating (132) we obtain

$$W = \frac{2GM}{c^2} + \left(\frac{\hbar c^2}{2mGM}\right)^2 \cdot \frac{1}{GM}(s - s_0)^2. \tag{134}$$

Formulas (132-134) coincide with the formulas from publication [1]. The presence of the Planck constant in equation (132) is connected with the wave nature of matter formed, in our case, of gravitons having a rest mass. From formula (134) it is apparent, that the test body can never intercept the Schwarzshild sphere. In GRT the situation is rather different. From the Schwarzshild solution and expression (129) it follows that the test body will cross the Schwarzshild sphere and a “black hole” will be formed. The test bodies or light can cross the Schwarzshild sphere only in the inside direction, thus they already can never leave the Schwarzshild sphere. We shall come to the same result if we proceed to a synchronous system of freely falling test bodies with the help of transformations

$$\tau = t + \int dW \left[\frac{\tilde{V}(1-U)}{U}\right]^{1/2} . \tag{135}$$

$$R = t + \int dW \left[\frac{\tilde{V}}{U(1-U)}\right]^{1/2} . \tag{136}$$

In this case interval (118) becomes

$$ds^2 = d\tau^2 - (1-U)dR^2 - W^2(d\Theta^2 + \sin^2\Theta d\Phi^2) . \tag{137}$$

In such a form singularities of metric coefficients dissapear both for the Schwarzshild solution, when $\epsilon = 0$, and for the solution in our case, when $\epsilon \neq 0$.

Subtracting from expression (136) expression (135) we get

$$R - \tau = \int dW \sqrt{\frac{U\tilde{V}}{(1-U)}} . \tag{138}$$

Differentiating equation (138) over τ we discover the following

$$\frac{dW}{d\tau} = -\sqrt{\frac{(1-U)}{U\tilde{V}}} . \tag{139}$$

Thus, we come to the same initial equation (129), around which the formulas (132-134) were obtained. Thus, it is abundantly clear that the transition to the synchronous falling reference frame does not eliminate the singularity which arises due to the presence of a graviton mass, i.e. when $\epsilon \neq 0$. In case when $\epsilon = 0$, the Schwarzshild singularity of the metric does not influence the motion of a test body both in the initial coordinate system and in the falling synchronous system. Thus, the falling particles cross the Schwarzshild sphere in the inside direction only.

Let us calculate now the propagation time for a light signal from a point W_0 up to the point $W_1 = \frac{2GM}{c^2}$. For the Schwarzshild solution from expression $ds^2 = 0$ we have

$$\frac{dW}{dt} = -c\left(1 - \frac{2GM}{c^2 W}\right) . \tag{140}$$

By integrating this equation we get

$$W_0 - W + \frac{2GM}{c^2} \ln \frac{W_0 - \frac{2GM}{c^2}}{W - \frac{2GM}{c^2}} = c(t - t_0) . \tag{141}$$

Hence it is apparent that to achieve the gravitational radius $W_1 = \frac{2GM}{c^2}$ in GRT we need an infinite time measured by a distant observer clock. In RTG, as we have established earlier, the Schwarzshild solution takes place up to the point $W = W_1(1+\lambda_2)$, and therefore the time interval to reach this point is equal to

$$c(t - t_0) = W_0 - W_1(1 + \lambda_2) + \frac{2GM}{c^2} \ln \frac{W_0 - \frac{2GM}{c^2}}{\lambda_2 \frac{2GM}{c^2}} . \tag{142}$$

The propagation time of a light ray from the point $W = W_1(1 + \lambda_2)$ up to the point W_1 can be computed by using formulas (97) and (108). In this interval we have

$$\frac{dW}{dt} = -c\frac{x_1^3}{2xf}(1 - \epsilon f). \tag{143}$$

Hence after integrating and replacement of a variable we get

$$\frac{2MG}{c^2} \int\limits_f^{1/\epsilon} \frac{xdf}{f} = c(t_1 - t) . \tag{144}$$

According to (84) and (108) the lower limit of integration is equal to

$$f = \tilde{C} = \frac{x_1^2}{2\lambda_2} . \tag{145}$$

Integral (144) is easily evaluated and with good accuracy results in the following relation:

$$c(t_1 - t) = W_1\lambda_2 + \frac{2GM}{c^2}\ln\frac{2\lambda_2}{\epsilon}\,. \tag{146}$$

On the basis of equations (142) and (146) the time needed for a light signal to pass the distance from the point W_0 up to the point $W_1 = \frac{2GM}{c^2}$ is equal to the sum of expressions (142) and (146)

$$c(t_1 - t_0) = W_0 - W_1 + \frac{2GM}{c^2}\ln\frac{W_0 - \frac{2GM}{c^2}}{\epsilon\frac{GM}{c^2}}\,. \tag{147}$$

Thus it is evident that in RTG, as opposed to GRT, the propagation time of a light ray up to the Schwarzshild sphere is finite also if measured by a distant observer clock. From formula (147) it is apparent that the propagation time of a signal does not sharply increase due to the gravitational field.

From the above it is apparent that in the presence of a graviton mass $\epsilon \neq 0$ the solution in RTG differs essentially from the Schwarzshild solution because of the presence of the Schwarzshild sphere singularity, which cannot be removed by any choice of coordinate system. For this reason, as we have shown above, the physical solution for a static spherically symmetric body is possible only in the case, when the point x_1 is inside the body. This conclusion is also preserved for the synchronous coordinate system, when the metric coefficients (see (134)) are functions of time.

Thus, according to RTG as a field theory of gravitation, the body of any mass cannot contract unlimitedly, and therefore the gravitational collapse to form a "black hole" is impossible. In GRT the energy release at a spherically symmetric accretion of matter on a "black hole" is not enough, as the falling matter carries energy into the "black hole".

According to RTG, the situation cardinally changes, as at the accretion the falling matter hits the surface of a body, and therefore the energy release is now considerable. The field approach to gravitation changes in essence our notions which were formed under the influence of GRT. In particular, this manifests in the fact that the effective Riemannian space which has arisen due to the gravitational field, has only simple topology, since the gravitational field in the Minkowski space, as well as any other physical field, can be described in a single Galilean coordinate system. In GRT the Riemannian space may have a complicated topology, and it is described by the atlas of charts.

Further it is noteworthy that the operation of a gravitational field, as well as any other physical field, does not move the trajectory of motion of a test body outside the causality cone of the Minkowski space. This circumstance allows one to compensate the three-dimensional gravitation force by a force of inertia through selection of an accelerated coordinate system.

There is a principal difference between gravitation forces and forces of inertia. The force of inertia can always be made equal to zero by having selected an inertial system of coordinates, whereas the gravitation force, which has arisen because of the presence of a gravitational field, is impossible to be made equal to zero by selection of a coordinate system, even locally.

If GRT asserts that the gravitation is the consequence of the space-time (Riemannian) curvature, then, according to RTG, the effective Riemannian space-time is a consequent of the presence of a gravitational field, possessing density of energy-momentum. The source of it is the energy-momentum tensor density of the entire matter, the gravitational field included. The space-time was and is the Minkowski space, and all the remaining, including the gravitation, are physical fields. Just under these notions the basic physical principles — the integral conservation laws of energy-momentum and angular momentum take place.

The field approach to gravitation with necessity requires the introduction of the graviton mass, which, in turn, makes the gravitational collapse impossible and results in the cyclical development of the homogeneous and isotropic Universe. Thus, the homogeneous and isotropic Universe is "flat", and the existence of "dark matter" in the Universe is a forecast [3]. It follows direct from the theory, that present density of matter in the Universe should be equal to

$$\rho(\tau) = \rho_c(\tau) + \rho_g, \tag{148}$$

where ρ_c is the critical density, determined by the Hubble "constant" $H(\tau)$ and is equal to

$$\rho_c = \frac{3H^2}{8\pi G}\,, \tag{149}$$

and ρ_g is determined by the graviton mass m and is equal to

$$\rho_g = \frac{1}{16\pi G}\left(\frac{mc^2}{\hbar}\right)^2. \tag{150}$$

Since critical density ρ_c many times exceeds the observable density of matter in the Universe, then, according to equation (148), there should be a dark matter in the Universe.

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Appendix A

In sperical coordinates of the Minkowski space the intervals of the Minkowski space and of the effective Riemannian spaces look like

$$d\sigma^2 = dt^2 - dr^2 - r^2(d\Theta^2 + \sin^2\Theta d\Phi^2) \ , \qquad (A.1)$$

$$ds^2 = U(r)dt^2 - V(r)dr^2 - W^2(r)(d\Theta^2 + \sin^2\Theta d\Phi^2) \ . \qquad (A.2)$$

Let us introduce the velocity vector

$$v^i = \frac{dx^i}{dt}, \ \ v^i = ve^i, \ \ (x^i = r, \Theta, \Phi) \ . \qquad (A.3)$$

where e^i is the unit vector defined by the metric of a spatial section of the Minkowski space-time

$$\kappa_{ik}e^ie^k = 1 \ . \qquad (A.4)$$

In general κ_{ik} is given as follows

$$\kappa_{ik} = -\gamma_{ik} + \frac{\gamma_{0i}\gamma_{0k}}{\gamma_{00}} \ . \qquad (A.5)$$

In case (A.1)

$$\kappa_{ik} = -\gamma_{ik}. \qquad (A.6)$$

Condition (A.4) for the metric (A.1) looks like

$$(e^1)^2 + r^2[(e^2)^2 + \sin^2\Theta \cdot (e^3)^2] = 1 \ . \qquad (A.7)$$

Let us define four-vector of velocity by the following equation

$$v^\mu = (1, ve^i) \tag{A.8}$$

and demand that it should be isotropic in the Minkowski space

$$\gamma_{\mu\nu} v^\mu v^\nu = 0 \ . \tag{A.9}$$

By substituting (A.8) into (A.9) and accounting for (A.7) we obtain

$$v = 1 \ . \tag{A.10}$$

Thus, isotropic four-vector v^μ is equal to

$$v^\mu = (1, e^i) \ . \tag{A.11}$$

As according to the Special Relativity Theory the motion always takes place inside or on the boundary of the Minkowski causality cone, the Principle of Causality takes place for the gravitational field

$$g_{\mu\nu} v^\mu v^\nu \leq 0 \ , \tag{A.12}$$

that is,

$$U - V(e^1)^2 - W^2[(e^2)^2 + (e^3)^2 \sin^2\Theta] \leq 0 \ . \tag{A.13}$$

By taking into account (A.7), expression (A.13) can be written as follows

$$U - \frac{W^2}{r^2}\left(V - \frac{W^2}{r^2}\right)(e^1)^2 \leq 0 \ . \tag{A.14}$$

Let

$$V - \frac{W^2}{r^2} \geq 0 \ . \tag{A.15}$$

By virtue of an arbitrariness of $0 \leq (e^1)^2 \leq 1$, inequality (A.14) will be fulfilled only if

$$U - \frac{W^2}{r^2} \leq 0 \ . \tag{A.16}$$

From inequalities (A.15) and (A.16) it follows that

$$U \leq V \ . \tag{A.17}$$

In case if

$$V - \frac{W^2}{r^2} < 0 \;, \tag{A.18}$$

we shall write inequality (A.14) in the following form:

$$U - V - \left(\frac{W^2}{r^2} - V\right)(1 - (e^1)^2) \leq 0 \;. \tag{A.19}$$

By virtue of the arbitrariness of e^1, (A.19) will be satisfied for any values of $0 \leq (e^1)^2 \leq 1$ only in case

$$U \leq V \;. \tag{A.20}$$

Thus, the RTG Principle of Causality results in all the cases in the inequality

$$U(r) \leq V(r) \;. \tag{A.21}$$